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IN
FOREST RESOURCES MANAGEMENT

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Introduction

In recent years forest resource managers have turned increasingly to the mathematics of optimization as an aid in making resource decisions. Many industrial organizations, for example, use linear programming as a harvest scheduling technique and specific computational procedures have been developed for this application (c.f., Ware and Clutter 1971). The Forest Service has routinely used linear programming for calculating potential yield and allowable sale quantities on the western National Forests (Navon 1971, Johnson and Scheurman 1971). Linear programming and related techniques have also been used or proposed for use in more general planning situations like unit planning or land use planning (Dress, *et al* 1977, Dress 1975, Bell 1977, Field *et al*).

The most common applications of quantitative methods in resources management have been in single criterion situations. In such situations the degree to which the objective of the manager is accomplished by a particular action or sequence of actions is assumed to be measured by a single variable, or criterion. Examples of criteria often used in measuring the worth of forest management actions are total volume harvested during the planning period, the net present value of all harvests

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[1]Associate Professor of Forest Resources, School of Forest Resources, University of Georgia, Athens, Georgia 30602.

[2]Operations Research Analyst, Southeastern Forest Experiment Station, USDA-Forest Service, Athens, Georgia 30602.

during the period, and the size of the sustainable harvest in either commodity terms or in terms of net economic worth. In any case, the choice of a suitable criterion depends on the manager's objective. Thus, if the manager's objective is profit maximization during the planning period, present net worth is usually assumed as the decision criterion and optimization methods are employed to select that management action which maximizes this criterion.

It has been realized that, except in nearly trivial cases, the objectives of management are not that simple. Even in the private sector, contrary to assumptions of neo-classical economic theory, producers are not generally strict profit maximizers. Their scope of action is usually highly constrained by risk levels, the desire to remain in a particular business sphere, limits on available resources, and the social, legal, and economic environment in which they operate. In public land management the legal requirements for sustained yield, multiple use, and compliance with environmental regulations clearly dictate that management objectives necessarily involve more than a single concern.

The recognition that management decisions, public and private, are often inherently more complex than can be accommodated by a single criterion variable has led to the development of analytical procedures for decision making under multiple criteria. This paper will focus on the multiple criterion decision problem by developing a framework for optimization when more than one criterion is involved. We will consider several multi-criterion optimization procedures, concluding with an example where these procedures are applied in a hypothetical multiple use situation.

Our intent in this paper is two-fold. First, we hope that discussions of the analytical methods, while by no means very complete or technically detailed, will illuminate the flavor of the methods used and will illustrate the technical and philosophical difficulties attendant with their use. Second, the application of this technology to a sample multiple use problem should illustrate very well the complexity of the analysis, the need to consider trade-offs in the analysis, and the need to make decisions outside the analysis through judgemental and political processes.

A Framework for Multiple Criterion Decisions

Although there is a general lack of consensus about the meaning of the word objective when used in a planning context, its meaning is fairly well specified in management science. We will take an objective to be a precise statement of a decision maker's intent in a situation where a decision problem exists; i.e., in a situation where a decision maker is faced with a choice among alternative courses of action in order to accomplish a desired end. We assume that an objective statement is sufficiently precise to permit quantification to the degree that one or more variables can be defined to measure the 'goodness' of each alternative course of action in terms of how effectively it accomplishes the objective.

In the special case where a single criterion variable is all that is required in the quantification of the decision maker's objective, ordinary mathematical optimization procedures are suitable. For example, suppose that the decision problem can be placed into standard linear programming form where the decision maker's alternatives can be represented by a set of independent variables x_1, x_2, ..., x_n. Constraints

on the selection of alternative courses of action can be expressed as a set of linear inequalities in the x_j, and the decision criterion, say z, can be expressed as a linear function of the x_j. The mathematically best decision would be obtained by solving the following linear program:

$$\max z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

subject to

$$\begin{aligned}
a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n &\leq b_1 \\
a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n &\leq b_2 \\
&\vdots \\
a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n &\leq b_m
\end{aligned}$$

and

$$x_j \geq 0;\ j = 1, 2, \ldots, n$$

In this formulation, the terms c_j, a_{ij}, and b_i are specified constant coefficients which can often by interpreted as follows:

i) c_j is the derived benefit (net return or utility) per unit of the jth activity (the management activity represented by the variable x_j);

ii) a_{ij} is the amount of resource i that will be used if management activity j is performed at the unit level; and

iii) b_i is the amount of resource i that is available to carry out this program.

In a typical forest planning situation, a_{ij} might be the amount of resource i (area of land, capital resources, labor, etc.) used per acre

of land allocated to use j, b_i the number of units of resource i available to carry out this allocation, and x_j the final number of acres of land allocated to land use j.

The solution of well-defined linear programming problems involves the selection of an optimal management alternative from the set of all alternatives which satisfy the constraints of the problem. The set of all $x_1, x_2, \ldots, x_n$ which satisfy the problem constraints is called the set of feasible alternatives or, more simply, the feasible set. It will be convenient for later discussion to represent this set by a single literal, say Ω. The alternative which maximizes z will be labeled x_1^*, $x_2^*, \ldots, x_n^*$. As a matter of definition, the function $c_1x_1 + c_2x_2 + \ldots + c_nx_n$ that serves to define z as a function of the x_j is called the objective function. The optimal value of z will be denoted by z^* where

$$z^* = c_1x_1^* + c_2x_2^* + \ldots + c_nx_n^*$$

Mathematical procedures to obtain optimal management alternatives from linear programming problems are well-known in management science and need not concern us here, except for the fact that they exist and can efficiently handle problems involving large numbers of independent variables and constraints.[3] As has been mentioned, linear programming

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[3]The general single-criterion variable mathematical programming problem can be written

$$\max z = g\ (x_1, x_2, \ldots, x_n),\ \text{subject to}$$

$$a_1(x_1, x_2, \ldots, x_n) \leq b_1$$

$$a_2(x_1, x_2, \ldots, x_n) \leq b_2$$

$$\vdots$$

$$a_m(x_1, x_2, \ldots, x_n) \leq b_m$$

procedures have been widely applied in forest resource management situations where objectives are singularly specified.

For the general case where two or more criterion variables are implied by the decision maker's objective, mathematical optimization procedures as usually applied will work only in the highly unlikely situation where all criteria are complementary rather than competitive. This, briefly put, means that an alternative that is chosen as best because it is optimal with respect to one criterion is also optimal with respect to all other criteria, a clearly unlikely situation in resources management where conflicting objectives are the rule.

The multiple criterion decision problem can be expressed in mathematical form as a straightforward notational extension of the standard linear programming problem. We will assume that appropriate linearity conditions hold, i.e., that constraints on alternative courses of action can be expressed in terms of a set of linear inequalities. In addition, we assume that the decision maker's objective implies p criterion variables, say $z_1, z_2, \ldots, z_p$, where

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[3] where $g(\cdot)$ and the $a_i(\cdot)$ are specified functions. Linear programming is simply the special case of this problem where $g(\cdot)$ and $a_i(\cdot)$ are linear functions of the activity variables x_j. While standard solution procedures exist for the linear case and certain other special cases (e.g., quadratic, geometric, integer programming) the general case is too general to permit any sort of standard treatment. In addition, programming problems other than linear are much more restricted in size by the efficiency of solution procedures. Fortunately for the sake of application, linear programming has been found to be quite adaptable and seems to accommodate well a great variety of resource management problems. We will confine our attention to the linear case and certain special variants in this paper.

$$z_k = c_{1k}\, x_1 + c_{2k}\, x_2 + \ldots + c_{nk}\, x_n,$$

the c_{jk} being specified constants. Thus, we are also requiring that the p decision criteria be linear functions of the <u>activity variables</u>.

The final statement of the multiple criterion decision problem requires that we deal with the cases of complementary and competing criteria separately. If the criteria are in fact complementary, then by definition there exists an optimal alternative $x_1^*, x_2^*, \ldots, x_n^*$ that is in Ω (i.e., that satisfies all program constraints) such that the optimal value of z_k, say z_k^*, is given by

$$z_k^* = c_{1k}\, x_1^* + c_{2k}\, x_2^* + \ldots + c_{nk}\, x_n^*$$

for all k; k = 1, 2, ..., p. This simply states mathematically that the same alternative is optimal for all criteria. For this case the optimization problem would take the form

$$\begin{aligned}
\max\ z_1 &= c_{11}x_1 + c_{12}x_2 + \ldots + c_{1n}x_n \\
\max\ z_2 &= c_{21}x_1 + c_{22}x_2 + \ldots + c_{2n}x_n \\
&\ \ \vdots \\
\max\ z_p &= c_{p1}x_1 + c_{p2}x_2 + \ldots + c_{pn}x_n
\end{aligned}$$

all subject to

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\le b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\le b_2 \\
&\ \ \vdots
\end{aligned}$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m$$

$$x_j \geq 0; 1, 2, \ldots, m.$$

Note that the set of constraints on management alternatives is the same for each criterion. Note also that this situation can be identified operationally by solving each of the p separate linear programming problems and determining whether or not the optimal solutions give identical values for the x_j^*.

For the more usual case where criteria are competing there will not exist a single solution that optimizes each of the p criteria simultaneously. Here we must recognize at the outset that the nature of the solution is fundamentally different than for the first case. Instead of seeking an optimal solution within the context of the model, we try to find the subset of alternatives from the set of all feasible alternatives that are in a clearly specified sense better than those that are not in this sub-set. To approach this formally we divide or partition the set of feasible alternatives Ω into two disjoint and exhaustive sub-sets, say Ω_1 and Ω_2. We will call Ω_1 the set of inferior (feasible) alternatives and Ω_2 the set of non-inferior (feasible) alternatives. The set Ω_2 is defined in terms of its elements as follows:

> an alternative $x_1', x_2', \ldots, x_n'$ is in Ω_2, the set of non-inferior strategies if there does not exist another alternative $x_1, x_2, \ldots, x_n$ in Ω such that
>
> $$z_k \geq z_k'$$
>
> for all values of k and
>
> $$z_k > z_k'$$

for at least one value of k. Here

$$z_k = c_{k1}x_1 + c_{k2}x_2 + \ldots + c_{kn}x_n,$$
$$Z_k' = c_{k1}x_1' + c_{k2}x_2' + \ldots + c_{kn}x_n'.$$

All alternatives which are not non-inferior are dominated by some other alternative in Ω and are in the set Ω_1, the inferior set.

Thus, all alternatives that are non-inferior are better in at least one respect than any inferior strategy.

An example may serve to clarify this definition. Suppose that the intent of management for a particular forest property is the production of wood fiber, water, and browse. Let z_1 be the amount of wood produced by a particular alternative, z_2 the amount of water, and z_3 the amount of browse by the same alternative. Alternatives here would consist of the various feasible allocations of land areas and other resources necessary to carry out management. Suppose that for two alternatives we find that production amounts are as follows:

Criterion	Alternative 1	Alternative 2
Wood	30 units	32 units
Water	2.5 units	2.5 units
Browse	1.5 units	1.6 units

Alternative 2 is clearly superior to alternative 1 since more wood and browse and a equal amount of water is produced by this alternative. Thus alternative 1 is an inferior strategy and need not be considered further. As a matter of terminology, alternative 1 is dominated by alternative 2. It should be noted that this simple comparison of two

alternatives does not insure that alternative 2 is non-inferior — there may be other feasible alternatives which dominate it. To show that an alternative is non-inferior it must be shown that there is no other alternative in Ω which dominates it.

Two points should be made concerning the notion of multiple criterion decision analysis. First, the set of non-inferior solutions is made up of alternatives that are mixed in the sense that they are not, on the basis of the values of the criteria, comparable. Suppose, to continue the preceding example, that we are able to demonstrate that alternative 2 is in fact, non-inferior. Suppose also that we find another non-inferior alternative and we consider the two alternatives in terms of their output levels:

Criterion	Alternative 2	Alternative 3
Wood	32 units	30 units
Water	2.5 units	3.0 units
Browse	1.6 units	1.6 units

Alternative 3 outperforms alternative 2 in water production, under performs in wood, and produces the same amount of browse. A choice of one of these alternatives for implementation must be based on criteria not included in the model. In this case, alternative 3 would be chosen only if it was deemed through extra-model analysis that the benefits associated with the gain in water production (0.5 units) exceeded the loss associated with the reduction in wood production (2 units). An alternative selected by any combination of analysis, judgment, and political process for implementation is called a <u>preferred alternative</u>. The usage of this term in management science is for all practical

purposes consistent with its use in planning and in law (c.f. NEPA 1969 42 U.S.C. 4321) although another term, best compromise solution, has been suggested (Belenson and Kapur 1973) and is perhaps more descriptive.

The second point concerning multiple criterion decision analysis follows from the first. Although there are a number of mathematical procedures for dealing with such problems, the selection of a final, preferred decision must be accomplished by somehow making the various objectives commensurate. This means that the decision maker explicitly or implicitly establishes a preference structure that ultimately orders solutions in the non-inferior set. If a total ordering of outcomes is available prior to analysis, i.e., if a utility function is available to value every combination of outputs in commensurate terms, then the decision problem reduces to a single criterion constrained optimization problem with this utility function taken as the objective function. Often, in multiple criterion problems, the utility function is not known in advance and a major purpose of analysis is to assist the decision maker in the development of a suitable expression of utility.

In order to develop an operational measure of utility in problems where there are competing criteria or objectives it would be desirable to:

i) know that alternatives under consideration are non-inferior;

ii) have explicit knowledge of the trade-offs between competing objectives over the full range of each objective; and

iii) know, in the case where criteria are expressed in natural units, e.g., board feet, animal unit months, tons, etc., the net value or utility of each unit of output over the full range of outputs.

For small problems, the computational burden required to obtain this information is often tolerable. But for problems as large in scale as the usual forest-level multiple use planning problem (by large here we mean large in terms of the number of criterion variables, in the number of activity variables, and in the number of constraints), the amount of computation required would be excessive and the amount of information that would have to be digested and ordered would be prohibitive. It is then necessary to employ compromise procedures that do not expose all non-inferior strategies and the associated trade-offs but take advantage of logical preference ordering and incomplete preference structuring by the decision maker in advance of the analysis to permit consideration of a greatly reduced set of alternatives. While it is possible using such procedures to overlook promising alternatives, one can at least be assured that the preferred alternative is non-inferior.

Mathematical procedures for multiple criterion decision analysis range from methods which identify or generate the entire set of non-inferior alternatives to those which seek to avoid the necessity for doing so. For the generating procedures, further analysis must be done to determine and evaluate trade-offs among objectives prior to the selection of a preferred alternative. Some of the procedures that attempt to circumvent the necessity for complete knowledge of the set of non-inferior stratagies are not appropriate in public decision making situations because they either assume that target output levels are specifiable when they are not or they require specific but inappropriate information from the decision maker in advance of knowledge of relevent trade-offs (Cohon and Marks 1975). Others, like goal programming, where optimization may proceed in a sequential manner over a set of

objectives which are to be satisfied in a specified priority order, can actually lead to the selection of inferior alternatives as preferred (Cohon and Marks 1975, Field 1978). There are some methods like the Surrogate Worth Analysis Technique (Haimes, et al 1975), Trade-off Development Methods (Goicoechea, et al 1976, 1979), and goal programming when used in appropriate combination with other optimization procedures (Field, et al), that do insure consideration of non-inferior alternatives and permit explicit evaluation of trade-offs for large scale problems. We will not go into these methods in any detail in this paper. The interested reader is referred to the papers previously cited for technical details and comparisons, particularly the papers by Haimes, et al (1975) and Cohon and Marks (1975).

A Forest Resource Planning Example

We will consider a hypothetical forest planning example in order to illustrate principles, methods of analysis, and pitfalls in dealing with multiple use as a multiple criterion decision problem. Suppose that we are interested in planning the long-term management of a forest property where the forest has been subdivided or classified into three sets of land parcels of uniform productivity. In terms of language now coming into use as a result of the regulations and Forest Service manual (sec. 1920) revisions attending the National Forest Management Act of 1976 (16 U.S.C. 1601-1614), individual homogeneous land parcels are called capability areas. Sets of capability areas that have been aggregated because all such areas are appropriately uniform in the scope, cost, and expected returns of planned management actions are called analysis areas. Our hypothetical forest is made up of three analysis areas and we will consider the planning problem in terms of deciding what sub-areas within these

analysis areas should be allocated to a set of long-term land uses. Decisions will be made only to the analysis area level, not to the capability area level. It is assumed that site-specific capability area allocation must be accomplished by combining the results of this analysis with a much more spatially-detailed analysis and this process will not be considered here — we will confine our interest to what is generally thought of as the planning decision, not the project scheduling decision.

We will also assume in this example that we are dealing with the steady state or sustainable yield part of the planning problem, not with the conversion to the steady state. To address the entire planning problem we would have to consider management both during the conversion period and during the period that begins when the desired steady state forest structure has been reached (Dress 1979). However, to simplify the example to a reasonable level we will address what might be termed the design question: "What kind of forest would we like to have in place (at the end of the conversion period) in order to maximize total net benefits available from sustained yield?" Simply posing this question presents certain philosophical difficulties that involve our ability to predict future preferences and even the desirability for doing so. We probably don't have anything more profound to say on this point than anybody else, but we will comment on this and other such difficulties following the example.

Suppose that we have decided on the basis of preliminary analyses to consider just four management alternatives that are described as follows:

Alternative 1 — establish an even-aged mixed upland hardwood forest and harvest by clear felling at age 100 years.

Alternative 2 — establish an uneven-aged mixed upland hardwood forest and manage by selection methods with a 10-year cutting cycle.

Alternative 3 — same as alternative 1 but with final harvest to be made at age 80.

Alternative 4 — place land under a management procedure that maintains an open condition (less than 10 trees per acre with a browse or forage groundcover).

We wish to determine the allocation of area to alternative land uses that "maximizes multiple use benefits." Suppose that the outputs that are of interest here are the same as those used in the earlier example: wood fiber, browse, and water, but suppose also that we wish to consider production costs as well. We will take as our translation of the multiple use goal, admittedly loose, that we desire to produce as much of each commodity as is possible at a low cost (note that these objectives are competitive).

To make matters more specific, we will assume the problem parameters given in Table I, approaching this problem in two ways:

i) as a multiple criterion problem where the intent is to generate the set of non-inferior alternative land allocations; and

ii) as a linear goal programming problem.

We will conclude the example with a comparative discussion of these two methods.

Table I. Relevant Data for the Forest Resource Planning Example (Data are Hypothetical)

Analysis Area	Size of Area	Alternative	Outputs for Alternative Courses of Action			Cost
			Wood	Water	Browse	
	Acres		Tons/Acre	Acre/Ft	Tons/Acre	$/Acre
1	3000	1	1.00	1.25	2.0	1.00
		2	0.80	1.00	3.0	1.25
		3	1.15	1.20	2.5	1.50
		4	0.00	1.40	4.0	25.00
2	5000	1	1.10	1.20	2.2	1.00
		2	0.85	1.05	3.2	1.25
		3	1.25	1.15	2.8	1.50
		4	0.00	1.50	5.0	35.00
3	7000	1	1.15	1.20	2.3	1.00
		2	0.90	1.05	3.3	1.35
		3	1.35	1.15	2.9	1.60
		4	0.00	1.50	6.0	40.00

The Generation Procedure. Generation procedures are aimed at developing the entire set of non-inferior land allocation alternatives. For this and subsequent formulations, we define activity variables x_{ij} as the number of acres allocated from analysis area i to land use j; i = 1, 2, 3, j = 1, 2, 3, 4. To permit compact presentation of the problem, let W_{ij} be the wood output per acre, H_{ij} be the water output per acre, B_{ij} be the browse output per acre, and C_{ij} be the cost per acre (natural units — tons, acre-ft., tons, and dollars assumed). We may then form four objective functions:

Wood $$z_1 = \sum_{ij}\sum W_{ij}\, x_{ij}$$

Water $$z_2 = \sum_{ij}\sum H_{ij}\, x_{ij}$$

Browse $\quad z_3 = \sum_{ij}\sum B_{ij}\, x_{ij}$

Cost[4] $\quad z_4 = -\sum_{ij}\sum C_{ij}\, x_{ij}$

As the problem is stated there are only three constraints; those insuring that no more or less than the total acreage in an analysis area be allocated. These constraints can be written:

$$\sum_j x_{1j} = 3000$$

$$\sum_j x_{2j} = 5000$$

$$\sum_j x_{3j} = 7000$$

We also have the usual non-negativity constraints

$$x_{ij} \geq 0.$$

To illustrate a generation technique we will consider only the objectives dealing with wood and water production and ignore those dealing with browse and cost. We do this only to permit a direct approach and to permit a graphical display of the set of feasible output combinations together with the non-inferior region. The feasible region in objective space is shown in Figure 1. The closed polygon shown in this figure is the set of points (z_1,z_2) which correspond to feasible land allocations from the three sites to the four specified treatments. If we consider the two points P_1 and P_2 we see that

[4] The criterion variable for cost is defined as $-\sum_{ij}\sum C_{ij}\, X_{ij}$ simply to permit the treatment of all objectives in a maximization sense. This is done only to simplify discussion.

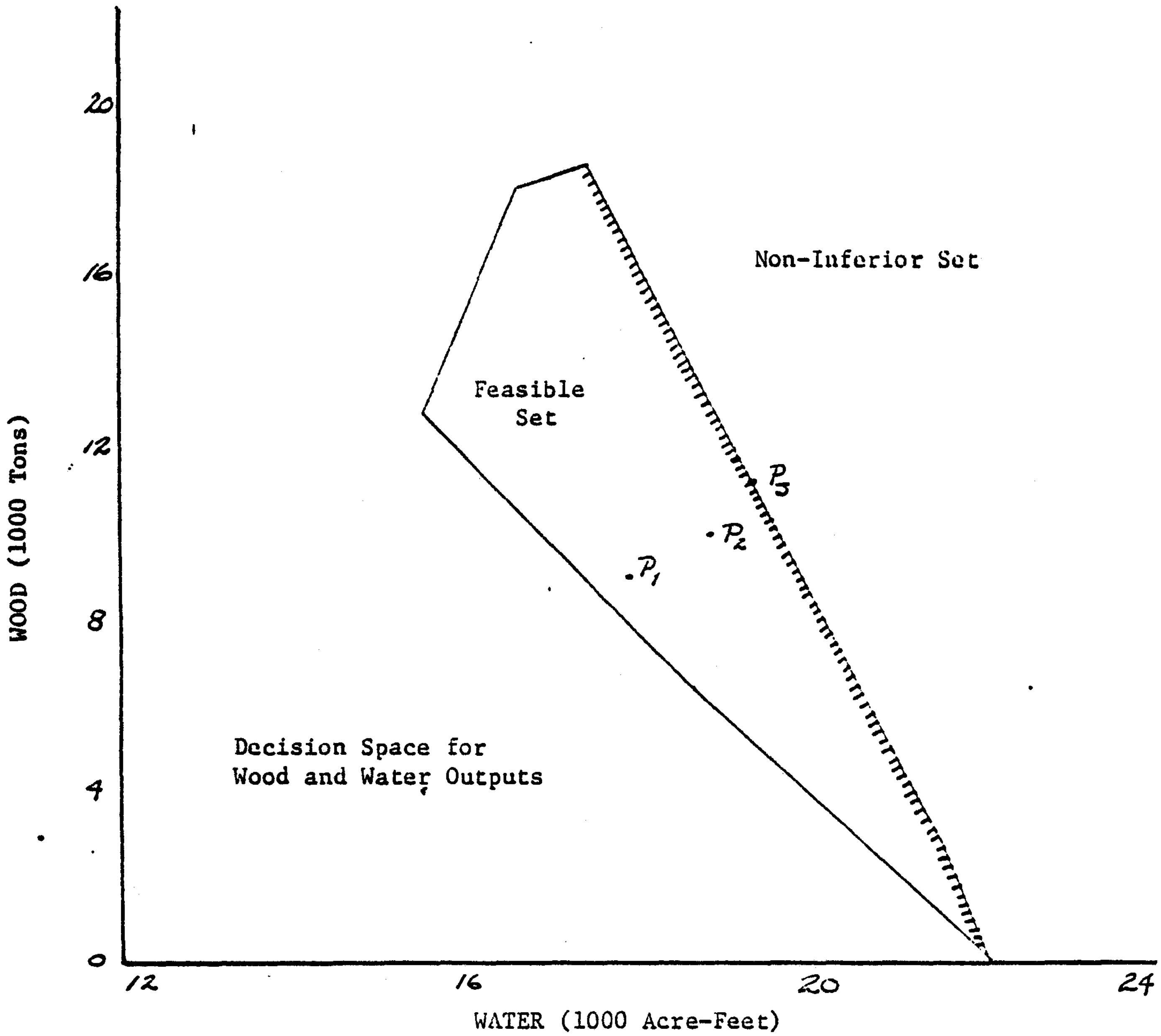

Figure 1. Set of Feasible Outcomes for Wood and Water for Reduced Multiple use Example.

Output	P_1	P_2
Wood	$z_1 = 9000$ Tons	$z_1 = 10000$ Tons
Water	$z_2 = 18000$ Acre-Feet	$z_2 = 19000$ Acre-Feet

so the allocation that gives rise to P_1 is clearly dominated by the allocation associated with P_2. But it is easy to see that P_2 is also inferior since we may increase z_1 and z_2 while remaining in the feasible region by moving up and to the right until the right-hand boundary of the set or polygon is reached. The point P_3, for example, corresponds to an allocation of land to treatments that would produce 11000 tons of wood and 19600 acre-feet of water. Since moving along this boundary (hatched in the figure) produces non-comparable strategies; i.e., we reduce wood output while increasing water output or vice versa, this boundary is the set of non-inferior outcomes. Having identified this set it would be necessary to bring additional preference information to bear to select a preferred outcome but information derived from the model can be used to assist the decision maker in developing such preference information. Of the greatest use in this regard are the trade-offs in production at given levels of production.

A legitimate question at this point is "Can we extend this method to more realistic problems?" Unfortunately, direct extensions, while possible for moderate-sized problems, are not generally possible for problems that interest resource managers (see Zadeh 1963, Cohon and Marks 1975). Our major purpose in presenting this simplified example was twofold:

i) it illustrates the abstract principles of earlier discussions; and

ii) the difficulty of extensions motivates a continuation of the example using methods which avoid the necessity for the complete enumeration of the set of non-inferior alternatives while assuring a consideration of only such alternatives in the selection of a preferred alternative.

A Goal Programming Approach

We now wish to consider the multiple use problem from the perspective of goal programming, a technique for multi-criterion decision making that avoids complete enumeration of the non-inferior set. As often used, goal programming will not insure that only non-inferior alternatives are considered as candidates for a best-compromise solution. The approach we employ here in setting goals at the level of their respective constrained optima does, however, guarantee this but only if all goals are treated at the same pre-emptive rank (Field 1978). This form of goal programming is called cardinally-weighted goal programming and, simply put, is a method for selecting alternatives that minimize the weighted sum of absolute deviations from stated goals, subject to problem constraints.

Mathematically, the cardinally-weighted linear goal programming problem can be stated as

$$\text{minimize } z = \sum_{k=1}^{q} w_k^- \, d_k^- + \sum_{k=1}^{q} w_k^+ \, d_k^+$$

subject to

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m$$

$$p_{11}x_1 + p_{12}x_2 + \ldots + p_{1n}x_n + d_1^- - d_1^+ = G_1$$

$$p_{21}x_1 + p_{22}x_2 + \ldots + p_{2n}x_n + d_2^- - d_2^+ = G_2$$

$$\vdots$$

$$p_{q1}x_1 + p_{q2}x_2 + \ldots + p_{qn}x_n + d_q^- - d_q^+ = G_q$$

$$x_j \geq 0;\ d_k^- \geq 0;\ d_k^+ \geq 0.$$

In this formulation G_k is the specified or target level of the kth criterion, the first m constraints are operational or ordinary problem constraints, and the last q equality constraints define the actual value attained by the q criterion variables and the deviations between actual levels and target levels. The d_k^- and the d_k^+ represent under-achievement and over-achievement of target or goal levels for the kth goal. Note that the deviational variables d_k^- and d_k^+ are defined so that either both are zero (the goal is achieved exactly) d_k^- is zero and d_k^+ is not (the goal is overachieved), or d_k^+ is zero and d_k^- is not (the goal is underachieved). The objective function is the weighted sum of these deviational variables where ideally weights are chosen to reflect the decision maker's preference for outcomes in terms of the importance of individual decision criteria. Small weights imply that over- or underachievement of a particular criterion is of less interest to the decision maker than a more heavily weighted criterion.

There are two major operational difficulties with goal programming:

i) specification of the goals or targets; and

ii) specification of the weights, the w_k^- and the w_k^+ that are associated with goal under- and overachievement.

In the example that follows we illustrate these difficulties, try to avoid them or at least minimize their effect, and attempt to illustrate a somewhat pragmatic planning procedure employing goal programming.

To make the example as previously stated somewhat more interesting for this application, we will introduce two additional features that are also of practical interest. First, we will assume that decision maker has decided on the basis of preference, prior analysis, and experience that browse levels are nearly adequate without the creation and maintenance of permanent openings and he would like no more than 5% of the forest so treated. We will also assume that he is interested in having a forest of some diversity and we wish to consider some way of measuring diversity and including this measure as one of the criteria by which management alternatives will be judged. The first requirement is easy to incorporate into the decision model -- we simply add to the area constraints the restriction that the area allocated from all sites to the openings treatment not exceed 5% of the total area or 750 acres,

$$x_{14} + x_{24} + x_{34} = \sum_{i=1}^{3} x_{i4} \leq 750$$

The second concern dealing with diversity is more difficult to handle and we will deal with it after considering the setting of goal levels for the other decision criteria and formulating the goal program for these criteria.

In order to insure that the goal programming formulation will deal only with non-inferior strategies it is necessary to set target levels at the optimum level for each criterion, given that other criteria are not considered. The determination of these levels can be accomplished by maximizing wood, browse, and water production and minimizing cost where each optimum is subject to the area and openings constraints. It is also of interest, for comparative purposes, to determine the worst possible outcomes for these criteria and this can be done by minimizing wood, browse, and water production and maximizing cost, again subject to the area and openings constraints. The individual maxima and minima for each criterion variable and the associated land allocations are shown in Tables II and III.

Using the results from individual optimization solutions we establish the following goals or targets for the goal programming formulation:

Wood -- G_1 = 19150 Tons (max)

Water -- G_2 = 18375 Acre-Feet (max)

Browse -- G_3 = 50125 Tons (max)

Cost -- G_4 = 15000 Dollars (min)

Since it is not possible to exceed the levels set for wood, water, and browse or to go below the cost goal, the goal program will require the minimization of

$$z = w_1^- d_1^- + w_2^- d_2^- + w_3^- d_3^- + w_4^+ d_4^+$$

subject to operational (area and openings) constraints and goal constraints (equations defining output levels, deviational variables and goal levels). If we assume that all deviations from goals are of equal

Table II. Individual Criterion Maxima and Associated Allocations for Multiple Use Example.

Output	Maximum Value	Allocation: Site 1 Treatment 1	2	3	4	Site 2 Treatment 1	2	3	4	Site 3 Treatment 1	2	3	4
Wood	19150	0	0	3000	0	0	0	5000	0	0	0	7000	0
Water	18375	3000	0	0	0	5000	0	0	0	6250	0	0	750
Browse	50125	0	3000	0	0	0	5000	0	0	0	6250	0	750
Cost	52000	0	0	3000	0	0	0	5000	0	0	0	6250	0

Table III. Individual Criterion Minima and Associated Allocations for Multiple Use Example.

Output	Minimum Value	Allocation											
		Site 1 Treatment				Site 2 Treatment				Site 3 Treatment			
		1	2	3	4	1	2	3	4	1	2	3	4
Wood	12275	0	3000	0	0	0	5000	0	0	0	6250	0	750
Water	15600	0	3000	0	0	0	5000	0	0	0	7000	0	0
Browse	33100	3000	0	0	0	5000	0	0	0	7000	0	0	0
Cost	15000	3000	0	0	0	5000	0	0	0	7000	0	0	0

importance to the decision maker and set the w_k^- and w_k^+ to values of 1.0, we obtain the solution given in Table IV. The allocation for this solution has all of the area in each of the three sites assigned to treatment 2, selection harvesting. A quick review of this solution would likely indicate two defects: no land is allocated to openings at all and some would probably be desired on the grounds of increased diversity and the under run on the timber goal is substantially more than on the other commodity targets.

Table IV. Attained Criterion and Criterion Target Levels for Multiple use Example Goal Program (all weights set to 1.0)

Criterion	Target Level	Attained Level	Deviation	
			Absolute	Percent
Wood	19150	12950	-6200	-32
Water	18375	15600	-2775	-15
Browse	50125	48100	-2025	- 4
Cost	15000	19450	+4450	+30

These apparent defects in the solution can be corrected to some degree by changing the weights assigned to deviations in order to more accurately reflect the decision maker's preference. Some iterative procedures for multi-criterion decision analysis (c.f., Surrogate Worth Trade-off Method (Haimes, et al 1975)) use the information in the optimal deviational solution to investigate trade-offs, elicit trade-off valuation from the decision maker, and develop trade-off value functions which permit approximate commensuration of previously non-commensurate criteria. These techniques have definite promise in forest

resource decision problems but they are somewhat complex in nature and we will not use that approach here.

We will attempt to arrive at what might be a more suitable solution by:

i) changing the goal formulation to specifically incorporate the decision maker's interest in diversity; and

ii) varying the weights in this modified solution to permit a more suitable statement of preference structure.

It should be pointed out, before addressing the diversity problem, that even though we have judged the goal solution obtained under equal weights to be unsatisfactory, it is nonetheless a non-inferior alternative. It should also be said that by judging or valuing the solutions from the program we are leaving, or at least expanding, the role usually occupied by the analyst. It is, of course, the responsibility of the decision maker to make such value judgements but for this example we assume that the analyst is also the decision maker.

The incorporation of diversity into the problem as a criterion for judging management alternatives presents a number of serious difficulties. First, there is the difficulty of measurement. There is not, nor is it likely that there can be, given the amount of information that must be conveyed, a completely satisfactory diversity index that applies in all situations. Second, even if we could agree on a suitable index of diversity (in fact there would be nothing to prevent the inclusion of several such measures as alternative criterion variables), the most suitable of such indices are non-linear functions of the probabilities or proportions of class membership (Pielou 1969). For the purposes of this example we will side-step these difficulties by proposing a very

simplistic solution. We will assume that an even subdivision of land among treatments would produce a forest which is suitably diverse. In the present case, maximum evenness would be attained if 750 acres (the maximum area permitted) is allocated to openings and the remaining area is evenly subdivided among the other three treatments. To address this in the decision model we introduce four additional targets,

G_5 = 4750 acres allocated to treatment 1,

G_6 = 4750 acres allocated to treatment 2,

G_7 = 4750 acres allocated to treatment 3, and

G_8 = 750 acres allocated to treatment 4.

The objective function for the goal program incorporating the evenness goals is

$$z = \sum_{k=1}^{q} w_k^- d_k^- + \sum_{k=1}^{q} w_k^+ d_k^+$$

where the last four d_k^- and d_k^+ represent deviations from the targeted area distribution. As before we minimize z subject to operational and goal constraints and, for equal weights on all deviations, the solution obtained in Table V is obtained. This solution has moved toward the wood and water targets at the expense of browse production and cost. The evenness requirement is reasonably well met except for the fact that no land is assigned to openings. It is easy to see that:

i) cost is limiting, to some degree, the area assigned to openings because of the high cost of treatment; and

ii) the target level for openings is small compared to other target levels and while the percent deviation is large (-100%), the absolute deviation is not.

Table V. Attained Criteria and Criterion Target Levels for Multiple Use Example with Diversity Considered (all weights set equal to 1.0)

Criterion	Target Level	Attained Level	Deviation Absolute	Deviation Percent
Wood	19150 T	16150	-3000	- 16
Water	18375 A-F	16787	-1587	- 9
Browse	50125 T	41450	-8675	- 17
Cost	$15000	18975	+3975	+ 27
Trt. Area 1	4750 A	4750	0	0
Trt. Area 2	4750 A	5500	+ 750	+ 16
Trt. Area 3	4750 A	4750	0	0
Trt. Area 4	750 A	0	- 750	-100

If we assume that the cost target, which has been set at the minimum, is somewhat unrealistic and is not of strict concern, and if we assume that some area in permanent openings is desirable, then the weight attached to deviations from the open treatment area target can be increased to reflect this preference. Suppose also that, because of this interest in land allocated to openings, we are willing to allocate additional funds to the budget and we raise the cost target from $15000 to $20000. Tables VI and VII show goal solutions obtained with the weight on open area underachievement set at 15.0 and 25.0 respectively.

The decision maker can employ data given in Tables V, VI, and VII to evaluate most of the important trade-offs associated with increasing the area in openings. Table VIII displays the trade-offs in commodity

Table VI. Attained Criteria and Criterion Target Levels for Multiple use Example with Diversity Considered (weight for underachievement of open area = 15.0, all other weights set to 1.0)

Criterion	Target Level	Attained Level	Deviation Absolute	Deviation Percent
Wood	19150 T	16123	-3026	-16
Water	18375 A-F	17050	-1324	- 7
Browse	50125 T	41482	-8642	-17
Cost	$20000	20000	0	0
Trt. Area 1	4750 A	4750	0	0
Trt. Area 2	4750 A	5467	717	+13
Trt. Area 3	4750 A	4750	0	0
Trt. Area 4	750 A	33	717	-96

Table VII. Attained Criteria and Criterion Target Levels for Multiple use Example with Diversity Considered (weight for underachievement of open area = 25.0, all other weights set equal to 1.0)

Criterion	Target Level	Attained Level	Deviation Absolute	Deviation Percent
Wood	19150 T	15550	- 3600	-19
Water	18375 A-F	17088	- 1288	- 7
Browse	50125 T	42200	- 7925	-16
Cost	$20000	36788	+16788	+84
Trt. Area 1	4750 A	4750	0	0
Trt. Area 2	4750 A	4750	0	0
Trt. Area 3	4750 A	4750	0	0
Trt. Area 4	750 A	750	0	0

Table VIII. Trade-offs in Outputs for 0 Acres, 33 Acres, and 750 Acres in Openings.

Criterion	0 vs 33 A	0 vs 750 A	33 vs 750 A
	Percent Differences		
Wood	0	- 3	- 3
Water	+ 2	+ 2	0
Browse	0	- 1	- 1
Cost	+ 33	+ 84	+ 51
	Absolute Differences		
Wood	0 T	- 600	- 576
Water	+ 263 T	+ 299	+ 36
Browse	0 T	+ 750	+ 700
Cost	+$5000	+21888	+16788

amounts for solutions obtained with diversity (evenness) considered and the cost target set equal to $20000. It is clear from an examination of these trade-offs that trade-offs in outputs are small and gains tend to balance losses. The increased cost of management is evident however and, in the final analysis, the decision maker must balance this added cost against gains toward the diversity goal. It would appear that we can go no further in the analysis to commensurate these quantities without additional preference information.

Two final remarks should be made concerning the analysis of this example. First, by raising the cost target from $15000 to $20000 we permit the possibility of the consideration of inferior alternatives

as candidates for a best compromise solution, but only in terms of the original problem. If, in fact, we are willing to make at least $20000 available, then alternatives considered with the cost target at that level are still non-inferior. Our last comment concerns the example _per se_. Since the example is entirely hypothetical no inferences can or should be drawn concerning the solutions of real-world multiple use problems. The procedures employed will obtain generally for such problems but extensive analysis is necessary in each case. Experience has shown that results are often counter-intuitive and highly sensitive to the vagaries of individual situations.

Summary and Conclusions

We have shown that the optimization methods customarily used to solve single-criterion problems can be extended to cases where more than one criterion should be used to evaluate management alternatives. Such cases certainly arise in multiple use management problems but in fact they often arise in situations which are commonly treated as single criterion problems. The extension in concept and methods required for addressing multiple criterion problems leads to three classes of solution procedures:

i) generating techniques where the entire set of non-inferior feasible alternatives is described;

ii) goal programming methods which in effect commensurate the proposed multiple criteria on the basis of a user specified utility function; and

iii) iterative techniques, which can also include goal programming, that use trial solutions to assist the decision maker

establish preference relations by explicit consideration of trade-offs at given levels of production.

An example was used to illustrate important theoretical concepts and to show the application of both generating methods and iterative goal programming.

A number of conclusions can be made on the basis of work reviewed in this paper. First, as has been shown by others (Cohon and Marks 1975, Goicoechea, et al 1979), procedures that generate the non-inferior set are not likely to be useful in the large-scale mathematical programming problems that arise in multiple use forest management. Such procedures may be useful when the number of criterion variables is small (two or three) and the number of constraints and activity variables is limited. Second, methods like goal programming when not used in an iterative manner to aid the decision maker in developing preference information are not likely to be really useful because they require explicit expression of priorities and weights in advance of solution. If this information is available, then an appropriate utility function can often be formulated and ordinary methods of optimization with all of their advantages can be employed instead of multiple criterion methods. Finally, if, as is usually the case in multiple criterion problems, the decision maker is not really aware of the trade-offs that he faces in arriving at a best compromise solution, iterative techniques that provide this information are likely to be the most suitable procedures. Among the procedures that are available for this purpose are iterative goal programming, particularly when used in conjunction with linear programming to establish targets or goals to insure non-inferiority, and surrogate worth trade-off methods. The latter methods are specifically

designed to make pairwise trade-offs among multiple criteria explicit and merit consideration for use in multiple use situations.

Literature Cited

Belensen, S. M. and K. C. Kapur. 1973. An Algorithm for Solving Multicriterion Linear Programming Problems with Examples. Operation Research Quarterly, (24) 1, 65-77.

Bell, E. F. 1976. Goal Programming for Land Use Planning. USDA Forest Service Gen. Tech. Rep. PNW-53. 12 p. Pacific NW For. and Range Exp. Stn., Portland, Or.

Cohon, J. L. and D. H. Marks. 1975. A Review and Evaluation of Multiobjective Programming Techniques Water Resources Research (11) 2, 208-220.

Dress, P. E. 1975. Forest Land Use Planning -- an Applications Environment for Goal Programming. P. 34-47 in Systems Analysis and Forest Resource Management. J. Meadows, et al, eds. Soc. Am. For., Bethesda, Md.

__________. 1979. A Two-Stage, Optimization Model for Forest Planning and Regulation. Report No. 2, Multifunctional Forest Planning Research Group, School of Forest Resources, University of Georgia, Athens, Georgia.

__________, K. D. Ware and L. S. Davis. 1977. A Land Management Planning Process for the National Forest System. Paper prepared for the NFMA Committee of Scientists. 50 pp. mimeo.

Field, R. C. 1978. Linear and Goal Programming as Complementary Planning Models for National Forest Timber Management. Unpub. Ph.D. dissertation, University of Georgia, Athens, Ga. 115 p.

__________, P. E. Dress, and J. C. Fortson. Complementary Linear and Goal Programming Procedures for Timber Harvest Scheduling. Accepted for Publication, For. Sci.

Goicoechea, A., L. Duckstein, and R. L. Bulfin. 1976. Multiobjective Stochastic Programming: the Protrade Method, pap. 76-18, Dept. of Sept. and Ind. Eng., Univ. of Ariz., Tucson.

__________, L. Duckstein, and M. Fogel. 1979. Multiple Objectives Under Uncertainty: An Illustrative Example of Protrade. Water Res. Res. (15) 2, pp. 203-210.

Haimes, Y. Y., W. A. Hall, and H. T. Freedman. 1975. Multiobjective Optimization in Water Resources Systems. Developments in Water Science 3, Elsevier Scientific Publishing Co., N. Y.

Johnson, K. N. and L. Scheurman. 1977. Techniques for Prescribing Optimal Timber Harvest and Investment Under Different Objectives -- Discussion and Synthesis. For. Sci. Mono. No. 18. 31 p.

Navon, D. I. 1971. Timber RAM: a Long-Range Planning Method for Commercial Timber Lands Under Multiple use Management. USDA For. Ser. Res. Paper PSW-70. 22 p. Pac. SW For. and Range Expt. Sta., Berkeley, Calif.

Ware, G. O. and J. L. Clutter. 1971. A Mathematical Programming System for the Management of Industrial Forests. For. Sci. 17:428-445.

Zadeh, L. A. 1963. Optimality and Non-scaler Valued Performance Criteria. IEEE Trans. Auto. Contr. AC-8(1), 59-60.